Theory of Transient Optical Nonlinearities in Semiconductor Microcavities

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Abstract—We present a theory of ultrafast time-resolved nonlinear spectroscopy in semiconductor microcavities including the influence of many-body and correlation effects beyond mean-field calculations. The theory, in close agreement with a number of experimental results, shows that semiconductor microcavities provide a unique tool for measuring the spectrum of four-particle correlations. We apply the theory to analyze the polariton parametric amplification in semiconductor microcavities. The obtained results demonstrate that exciton–exciton collisions can be controlled and engineered to produce almost decoherence-free collisions for the realization of completely optical microscopic devices.

1. INTRODUCTION

Manipulating the interaction between light and semiconductors by engineering both the electron and photon states has enabled a variety of striking coherent phenomena to be observed in recent years. A semiconductor microcavity (SMC) is a photonic structure designed to enhance light–matter interactions. Cavity-polaritons are two-dimensional eigenstates of SMCs resulting from the strong resonant coupling between cavity-photon modes and two-dimensional excitons in embedded quantum wells (QWs) [1]. The dynamics and hence the resulting energy bands of these mixed quasiparticles are highly distorted wiht respect to those of bare excitons and cavity photons (Fig. 1). The exciton–photon coupling rate V determines the normal mode (or Rabi) splitting ($2V$) between the two polariton energy bands. Extensive studies of semiconductor microcavities (SMCs) have shown normal mode splitting of polaritons in reflection and emission spectra and normal mode oscillations in transient optical response. These peculiar light–matter waves can be directly excited and manipulated by ultrafast laser pulses seeding the SMC at specific incidence angles. Moreover, the exciton and photon content of these mixed quasiparticles can be varied since it depends on the excitation angle and on the energy difference between the cavity mode (in the absence of QWs) and excitons in the bare QW.

The ultrafast nonlinear optical response of SMCs is attracting growing interest for exploring fundamental light–matter interactions in many-particle quantum systems, as well as for future implementation of ultrafast photonic devices and of devices based on quantum correlations. The recently observed ultrafast phase-coherent amplification of light–matter waves (cavity-polaritons) is one of the most exciting developments in the field of semiconductor nonlinear optics [2–11]. It operates with analogous principles (but under very different conditions) of matter-wave amplifiers based on ultracold atoms [12, 13]. Very recently, it has been shown that a semiconductor microcavity (SMC) can amplify (via phase-coherent amplification of polaritons) a weak light pulse by more than 5000 times [11].

Here, we present a microscopic theory of the time-resolved nonlinear response of SMCs in the coherent regime. The theory includes exciton–exciton collisions beyond mean-field calculations and shows that (i) SMCs provide a unique tool for use in investigating these many-body correlations and that (ii) correlation effects have a very important impact on the transient nonlinear response and on parametric polariton amplification. Although the mean-field theory has enjoyed

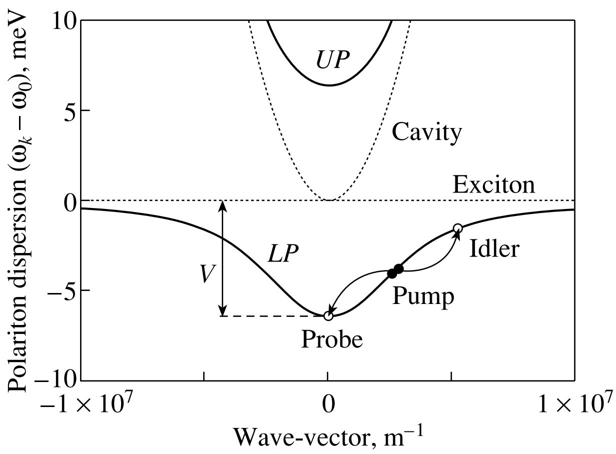

Fig. 1. Sketch of the polariton energy versus the in-plane wavevector for an SMC with splitting $2V$ = 10.4 meV. A polariton mode of given wavevector k can be excited by resonant light with incidence angle θ given by the relation $k = (\omega/c)\sin\theta$. UP and LP denote the split upper polariton and lower polariton curves. The dotted lines describe the dispersion curves for uncoupled excitons and cavity photons. The parametric polariton–polariton scattering process described in Section 5 is also depicted.

considerable success [14] in describing the nonlinear optical properties of semiconductors, recent studies revealed that collisions between excitons are more complex (see [15] and references therein) and involve multiparticle (at least four-particle) entangled states. This complexity, induced by the Coulomb interaction between electrons, determines the finite duration of collisions [15–21]. We find that the strong coupling of excitons with cavity-photons, giving rise to polaritons, alters the excitonic dynamics during exciton–exciton collisions, producing a modification of the effective scattering rates. As a consequence, the coherent nonlinear response of polaritons can be rather different from that of excitons in bare QWs and can depend significantly on the exciton–photon coupling rate V. In the following we will show that these differences (ignored by mean-field calculations) are at the basis of the giant parametric amplification uncovered in SMCs.

2. THE EQUATIONS OF MOTION

The system that we investigate consists of one or more QWs grown inside a wavelength scale semiconductor planar Fabry–Perot. We start from the equations for the third-order exciton polarization and cavity field describing quantum optical effects and coherent nonlinear optics in semiconductor microcavities [17] and perform the semiclassical factorization. Then, we follow the procedure outlined by Östreich *et al.* in [16], where the nonlinear term coming from Coulomb interaction was expressed as an exciton–exciton (X–X) mean-field interaction plus a correlation term expressed as a two-exciton correlation function. By inspecting all the steps of their derivation, it is possible to demonstrate that this procedure can also be applied to the nonperturbative regime of [17, 18]. This theoretical scheme is based on the dynamics-controlled truncation scheme (DCTS) [22], and calculations have been performed in the so-called coherent limit, which is justified as we are interested in the early time regime where the exciton dynamics is mainly coherent. The time evolution of the coupled exciton ($P_{\mathbf{k}}$) and photon waves ($E_{\mathbf{k}}$), including a finite duration of exciton–exciton collisions [18], can be described by the following set of coupled equations:

$$\begin{aligned}\frac{\partial}{\partial t}E_{\mathbf{k}} &= -(\gamma_c + i\omega_k^c)E_{\mathbf{k}} + iVP_{\mathbf{k}} + t_c E_{\mathbf{k}}^{\text{in}},\\ \frac{\partial}{\partial t}P_{\mathbf{k}} &= -(\gamma_x + i\omega_x)P_{\mathbf{k}} + iVE_{\mathbf{k}} - i\Omega_{\mathbf{k}}^{NL},\end{aligned} \tag{1}$$

where ω_k^c, ω_x and γ_c, γ_x are the energies and dephasing rates of cavity photons and QWs excitons, respectively. $E_{\mathbf{k}}^{\text{in}}$ describes input light pulses, and t_c determines the beam fraction passing the cavity mirror. The intracavity photon field and the exciton field of a given mode $\mathbf{k}$ are coupled by V. The polariton splitting in SMCs can be increased by inserting a large number of QWs into the cavity ($V = V_1\sqrt{N_{\text{eff}}}$, where V_1 is the exciton–photon coupling for one QW and the effective number of QWs N_{eff} depends on the number of wells inside the cavity and their spatial overlap with the cavity mode). The relevant nonlinear source term, able to couple waves with different in-plane wavevector $\mathbf{k}$, is given by $\Omega_{\mathbf{k}}^{NL}$ = $(\Omega_{\mathbf{k}}^{\text{sat}} + \Omega_{\mathbf{k}}^{xx})/N_{\text{eff}}$, where the first term originates from the phase-space filling of the exciton transition

$$\Omega_{\mathbf{k}}^{\text{sat}} = \frac{V}{n_{\text{sat}}}\sum_{\mathbf{kk'}} P_{\mathbf{q}}^* P_{\mathbf{k''}} E_{\mathbf{k'}}, \tag{2}$$

with $n_{\text{sat}} = 7/(16\pi a_0^2)$ being the exciton saturation density (a_0 is the exciton Bohr radius) and where $\mathbf{k}'$, $\mathbf{k}''$, and $\mathbf{q}$ are tied by the momentum conservation relation $\mathbf{k} + \mathbf{q} = \mathbf{k}' + \mathbf{k}''$. $\Omega_{\mathbf{k}}^{xx}$ is the Coulomb interaction term, which dominates the coherent exciton–exciton coupling and for cocircularly polarized waves can be written as

$$\begin{aligned}\Omega_{\mathbf{k}}^{xx} &= \sum_{\mathbf{k'k''}} V_{xx} P_{\mathbf{q}}^*(t) P_{\mathbf{k''}}(t)\\ &\quad - iP_{\mathbf{q}}^*(t)\int_{-\infty}^{t} F(t-t')P_{\mathbf{k''}}(t')P_{\mathbf{k'}}(t'),\end{aligned} \tag{3}$$

where $\Omega_{\mathbf{k}}^{xx}$ includes the instantaneous mean-field exciton–exciton interaction term $V_{xx} \simeq 6.08 E_b a_0^2$ (E_b is the exciton binding energy) plus a noninstantaneous term originating from four-particle correlations. This coherent memory can be interpreted as a nonmarkovian process involving two-particle (excitons) polarization waves interacting with a bath of four-particle correlations [15, 16] (see Eq. (4) below). We observe that the strong exciton–photon coupling does not modify the memory kernel $F(\tau)$ as a consequence of the fact that four-particle correlations do not couple to cavity photons [17–19, 21]. However, cavity effects are able to alter the phase dynamics of the tow-particle polarization waves $P_{\mathbf{k}}$ during collisions, i.e., on a timescale shorter than the decay time of the memory kernel $F(\tau)$. In particular, the phase of two-particle polarization waves in SMCs oscillates with a frequency (fixed by the polariton dispersion relations) modified with respect to that of excitons in bare QWs. This fact produces a modification of the integral in Eq. (3). According to quantum mechanics, collisions between exciton waves can be regarded as interference phenomena. The strong-coupling regime of SMCs, which alterss the phase of the interacting waves, provides a means to control the interference process. In this way, the exciton–photon coupling V affects the exciton–exciton collisions that govern the polariton amplification process. In this way, the exciton–photon coupling V affects the exciton–

exciton collisions that govern the polariton amplification process. The inset to Fig. 2 displays the memory kernel $F(\tau)$ for two-dimensional excitons. We calculated $F(\tau)$ following a recent microscopic approach [19, 20] based on the T matrix (Fig. 2a). Figure 2 also displays the complex quantity $\mathcal{F}(\omega)$ given by the Fourier transform of the memory kernel $F(\tau)$ plus the mean-field contribution V_{xx}. The imaginary part of $\mathcal{F}(\omega)$ describes the density of states (as a function of energy) of four-particle states weighted by their overlap with two-particle states (the excitons) and can be written [16], in the limit of zero homogeneous line broadening, as

$$\mathcal{F}(\omega) = \sum_m \left|\langle 0|\hat{D}|E_m\rangle\right|^2 \delta(\omega - \omega_m), \quad (4)$$

where $\hat{D} = [\hat{B}, [\hat{B}, \hat{V}_C]]$ ($\hat{B}$ is the exciton destruction operator, while V_C is the Coulomb interaction operator) and $|E_m\rangle$ and ω_m describe the eigenstates and the corresponding continuous spectrum of energies of the four-particle Hilbert subspace.

3. WEISSKOPF–WIGNER THEORY OF TWO-EXCITON CORRELATIONS

The influence of polariton effects on exciton–exciton collisions can be better understood by approximating the integral in Eq. (3) describing many-body effects beyond mean-field calculations. Let us consider a situation where the energies of the exciting pulses are all close to the corresponding polariton resonance values ω_k and the broadening is small as compared to the polariton splitting $2V$. Moreover, we assume zero time delay between the light pulses seeding the SMC. Then, it is a good approximation to replace the integral in Eq. (3) with a simpler expression, adopting the well-known Weisskopf–Wigner approximation used to analyze the spontaneous emission between two atomic levels. Within this approximation, the dominant exciton–exciton interaction term can be written as

$$\Omega_{\mathbf{k}}^{xx} = P_{\mathbf{q}}^*(t) \sum_{\mathbf{k}', \mathbf{k}''} \mathcal{F}(\omega_{\mathbf{k}'} + \omega_{\mathbf{k}''}) P_{\mathbf{k}''}(t) P_{\mathbf{k}'}(t). \quad (5)$$

This Weisskopf–Wigner description of four-particle correlation effects in SMCs distinguishes the relevance of polariton pairs in the scattering process. The energy of these pairs determines the effective scattering rates. The obtained four-particle spectral density (see Fig. 1) displays strong variations within the spectral region of interest around $2\omega_0$. In particular, moving towards the low-energy region, the dispersive part $\mathrm{Re}(F)$ increases while the absorptive part $\mathrm{Im}(F)$ that contrasts gain goes to zero. As a consequence, the exciton–exciton scattering rates in SMCs differ significantly from those of resonantly excited excitons in bare QWs. Mean-field calculations do not take into account this difference.

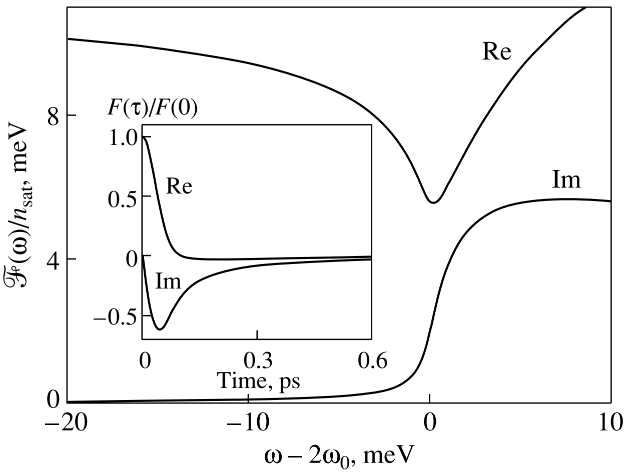

Fig. 2. Energy dependence of the effective exciton–exciton scattering potential. This is the sum of an energy independent term (the mean-field contribution) plus a term originating from four-particle (two electrons and tow holes) states of the QW. It has been calculated (following the microscopic approach described in [15]) for a GaAs QW 7 nm wide with an exciton binding energy of 13.5 meV and using, as the dephasing rate of the four-particle states, $\Gamma = 2\gamma_x$ = 0.58 meV (corresponding to a temperature T = 10 K). The tail of $\mathrm{Im}(\mathcal{F}(\omega))$ at negative detuning ($\omega < 2\omega_0$) is produced by Γ and vanishes for $\Gamma \longrightarrow 0$. The inset shows the dynamics of the memory kernel $F(\tau)$.

4. PUMP AND PROBE

Let us consider (1) a weak signal beam (2) and a more intense pump of ultrafast light pulses seeding the SNMC with zero time delay, each spectrally centered on the lower or upper polariton curve. We consider a specific configuration with the probe beam sent at normal incidence (along the growth axis of the SMC) and a slightly tilted ($\mathbf{k} \simeq 0$) pump beam with the same circular polarization of the probe. We also assume that the two pulses are sufficiently spectrally narrow to excite only one normal mode (the lower at ω_L or the upper at ω_U). We can thus apply the Weisskopf–Wigner approximation described in the previous section. We observe that the adopted configuration, owing to the sharpness of the polariton dispersion around $\mathbf{k} = 0$, does not allow the simultaneous satisfaction of energy and momentum conservation for the generation of additional light beams. From Eq. (7) the relevant exciton–exciton interaction term for the signal mode is given by

$$\Omega_1^{xx} = \mathcal{F}(\omega_1 + \omega_2)|P_2(t)|^2 P_1(t). \quad (6)$$

According to this equation, the presence of the pump produces an intensity-dependent shift ($\Delta_{NL} = \mathrm{Re}\mathcal{F}(\omega_1 + \omega_2)|P_2(t)|^2$) and a nonlinear absorption rate ($\Gamma_{NL} = \mathrm{Im}\mathcal{F}(\omega_1 + \omega_2)|P_2(t)|^2$) of the polariton resonance excited by the probe beam. These transient nonlinear effects originate from the self-scattering of a polariton pair comprising a pump polariton at ω_2 and a probe polariton at ω_1, and the scattering rate depends on the pair energy $\omega_1 + \omega_2$. The dependence of the resonance shift and of nonlinear absorption on the pair energy is a direct consequence of the noninstantaneous character of exciton–exciton collisions. In particular, Γ_{NL} is directly proportional to the spectral density of the equal-spin correlation function $\mathrm{Im}\mathcal{F}(\omega)$ calculated at $\omega = \omega_1 + \omega_2$. We observe that by tuning the relative energy Δ between cavity and exciton resonances, and in addition exploiting the three possibilities $\omega_1 + \omega_2 = 2\omega_U$, $\omega_U + \omega_L$, and $2\omega_L$, it is possible to span Γ_I and hence $\mathcal{F}(\omega)$ over a spectral region centered on $2\omega_0$ that is larger than 40 meV for typical III–V SMCs. This analysis thus shows that routinely measured transient signals, such as those from pump and probe experiments on SMCs, can provide direct information on the spectrum of four-particle correlations. We also observe that this Weisskopf–Wigner theory of exciton–exciton collisions, together with the microscopic calculation of the spectral function $\mathrm{Im}\mathcal{F}$, clearly predicts a suppression of the power-dependent absorption for $\omega_1 + \omega_2 < 0$ (see Fig. 1). For a typical SMC in the nonperturbative regime and at zero or negative detuning, this behavior predicts a suppression of the power-dependent absorption when both the pump and the probe are tuned on the lower polariton branch ($\omega_1 + \omega_2 = 2\omega_L$). This suppression is of great importance for many applications. In particular, it is expected to favor the giant amplification of lower polaritons [2]. This suppression was first observed by Baumberg *et al.* in [23] and has also been observed by other groups [24–27] on different samples and by adopting different experimental techniques. Good quantitative agreement of this theory with experimental results was demonstrated recently [18].

5. POLARITON PARAMETRIC AMPLIFICATION

It has recently been shown that increasing the exciton–photon coupling V (inserting a large number of QWs into the cavity) greatly increases amplification [11], thus allowing the design of several possible microscopic devices approaching room-temperature operation. This spectacular enhancement of gain contrasts with the results of present theories [28, 29] describing the process. According to these descriptions, the effective polariton–polariton interaction does not depend on the polariton splitting ($2V$), as it is the exciton content of three interacting modes independent of V. From this observation and from a direct inspection of the pertinent equations, it follows that increasing the exciton–photon coupling should not increase the amplification of a weak signal beam, which is in contrast with the spectacular experimental observations of [11].

Here, we show that the great enhancement of gain, observed when increasing the polariton splitting, origins from the unique interplay of the noninstantaneous nature of exciton–exciton interactions [15] with the strong coupling regime giving rise to polaritons.

Coherent amplification of polaritons requires a coupling mechanism able to transfer polaritons from a reservoir (in this case provided by polaritons resonantly excited by a pump laser pulse on the lower polariton dispersion) to the signal mode while conserving energy and momentum. Polaritons of different modes are coupled via their excitonic content, the coupling being provided mainly by the Coulomb interaction between excitons (nonlinearities arising from phase-space filling provide a minor contribution). This effective polariton–polariton interaction scatters a pair of pump polaritons into the lowest energy state and into a higher energy state (usually known as the idler mode) (energy and momentum conservation requires that $2\omega_k = \omega_0 + \omega_{2k}$, where ω_k is the energy of a pump polariton injected with an in-plane wavevector $\mathbf{k}$), as shown in Fig. 1. The scattering process is stimulated by a weak signal beam injected perpendicular to the cavity ($\mathbf{k} = 0$), thus greatly amplifying it. This process can be described by the set of equations (1) limited to the three interacting modes and with E^{in} different from zero only for the signal and the pump modes. By adopting once again the Weisskopf–Wigner approximation, the dominant exciton–exciton interaction term for the signal (0) and idler ($2\mathbf{k}$) modes can be written as

$$\begin{aligned}\Omega^{xx}_{0(2\mathbf{k})} &= \mathcal{F}(\omega_{0(2\mathbf{k})} + \omega_{\mathbf{k}})|P_{\mathbf{k}}(t)|^2 P_{0(2\mathbf{k})}(t) \\ &\quad + \mathcal{F}(2\omega_{\mathbf{k}})P^*_{2\mathbf{k}(0)}(t)P^2_{\mathbf{k}}(t).\end{aligned} \tag{7}$$

The first term (arising from self-scattering of polariton pairs) is analogous to that discussed in the previous section; it produces a blue shift of the polariton resonance and introduces an intensity-dependent dephasing mechanism. The second term provides a coupling mechanism able to transfer polaritons from the pump to the signal and idler modes. This term describes the scattering of a polariton pair (two pump polaritons) into a different pair (the signal plus the idler) with pair energy and momentum conservation. An analogous expression can be derived for the exciton–exciton interaction of the pump mode. This Weisskopf–Wigner description of four-particle correlation effects in SMCs sets out the relevance of polariton pairs in the amplification process. The energy of these pairs determines the effective scattering rates. As already discussed, the obtained four-particle spectral density displays strong variations within the spectral region of interest around $2\omega_0$. In particular, moving towards the low energy region, the absorptive part $\mathrm{Im}(F)$ that contrasts gain goes to zero. We observe that the energies of the pump, signal, and idler polaritons lower with an increase in polariton splitting ($2V$) (see Fig. 1a). The increase in V thus modifies the effective collision rates in such a way that the amplification process is favored. This analysis explains the large increase of gain observed when maximizing the exciton–photon coupling [11]. Moreover, it shows that in SMCs, it is possible to produce almost decoherence-free polariton–polariton collisions on the lower polariton curve by increasing the polariton splitting $2V$.

On the contrary, collision-induced decoherence increases for light–matter waves on the upper polariton curve. However, polaritons on the upper curve are not involved in the amplification process.

The microscopic theory of polariton parametric amplification presented here allows a fully quantitative analysis of recent measurements. In particular, we calculate the integrated gain in transmission as a function of the pump-pulse power (expressed in photons/cm^2). The gain curves, obtained by numerically solving the system of integrodifferential equations (1) for GaAlAs-based samples with splitting $2V_1$ = 10.6 meV and $2V_2$ = 15 meV (Fig. 3a), fully confirm this analysis. An increase of less than a factor 1/3 in the polariton splitting produces an increase in the maximum achievable gain at T = 10 K (Fig. 3a) of more than one order of magnitude (Fig. 3a), which is in close agreement with the experimental results obtained in [11]. The power dependence of gain (Fig. 3) shows an almost exponential growth and then saturates at high powers [31]. The saturation of gain is mainly determined by the nonlinear absorption, which is most relevant for the idler beam (according to Eq. (4), the idler nonlinear absorption is determined by $\mathrm{Im}\mathcal{F}(\omega_{2\mathbf{k}} + \omega_{\mathbf{k}})$). We observe that the increase in the idler nonlinear absorbance is highly superlinear, because the increase in the pump power produces both a direct increase in the exciton density and an increase in $\mathrm{Im}\mathcal{F}(\omega_{2\mathbf{k}} + \omega_{\mathbf{k}})$ as a consequence of the blue shift of the polariton-pair resonance $\omega_{2\mathbf{k}} + \omega_{\mathbf{k}}$ induced by $\mathrm{Re}\mathcal{F}$. We observe that mean-field calculations (also reported in Fig. 3a) overestimate the experimentally observed gain by more than four orders of magnitude; in addition, they are not able to reproduce the strong dependence of gain on the exciton–photon coupling V, which is a direct consequence of the inner dynamics of exciton–exciton collisions. Such a dramatic failure of the mean-field theory is surprising and clearly shows the technological relevance that many-body correlations and their optical control may assume in the future development of semiconductor completely optical devices. The gain curve (not shown here) obtained for a structure with only one QW ($2V$ = 5.3 meV) does not reach values of 20, which is in agreement with experimental observations from [9] and in contrast with mean-field calculations. This demonstrates that a large polariton splitting is essential to ensure collisions with low decoherence and hence significant amplification. Figure 3b, which displays results for T = 77 K, shows how the increase in exciton–photon coupling favors high-temperature operation. At T = 77 K, sample V_1 is still able to sustain a significant gain about six times larger than that of sample V_2.

The low-temperature time dynamics of the parametric amplification is displayed in Fig. 4. The appearance of an additional light beam in a direction allowing momentum conservation (idler mode), also observed experimentally in [2, 9], confirms the coherent nature of this scattering process involving pairs of polariton waves. Analogous results (not shown) but with a faster decay of the signals, due to the increased exciton decay rate, are found at higher temperature. The smallness of emission in the idler direction (as compared to the amplified signal) is a consequence of the low photon content of polaritons at high angles.

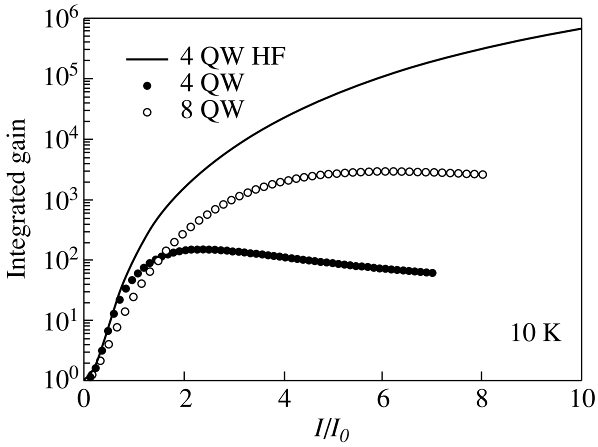

Fig. 3. The power dependence of the integrated gain (the total light intensity transmitted in the signal direction divided by the intensity transmitted in the absence of the pump beam) calculated for two GaAlAs-based samples with splitting $2V_1$ = 10.6 meV (corresponding to N_{eff} = 4) and $2V_2$ = 15 meV (N_{eff} = 8). I_0 = 10^{13} photons per cm^2 per pulse. The exciting light pulses have a duration of 250 fs. The material parameters of the two samples are coincident except for N_{eff}. The decay rate of cavity photons through the mirrors γ_c = 0.25 meV is the same as that of GaAlAs structures considered in [11]. The 7 nm wide QWs have a binding energy E_b = 13.5 nm. The homogeneous exciton broadening, mainly due to exciton–phonon scattering, depends on temperature. Calculations have been performed by using homogeneous broadenings extracted from polariton linewidth measurements in GaAlAs structures [30], $2\gamma_x$ = 0.6 meV (T = 10 K). The gain curves have been obtained by fully optimizing incidence angles, central frequencies of the input light beams, and energy of the cavity mode. Maximum gain occurs when the energy of the cavity mode is lower than the exciton energy by slightly more than half the polariton splitting.

6. CONCLUSION

We have investigated the role of four-particle correlations in the ultrafast time-resolved nonlinear response of SMCs. We have shown that they can significantly affect the transient nonlinear optical properties of SMCs. We have clarified the origin of the observed very different nonlinear absorption rates of the two-polariton branches, showing that they reflect the spectral properties of the two-exciton scattering continuum. Furthermore, the analytical study and the numerical results presented demonstrate that the nonperturbative regime of SMCs provides a unique tool for accessing the spectrum of four-particle correlations. Finally, we observe that the wavelike nature of excitations in solids tells us that collisions between excitons are nothing but

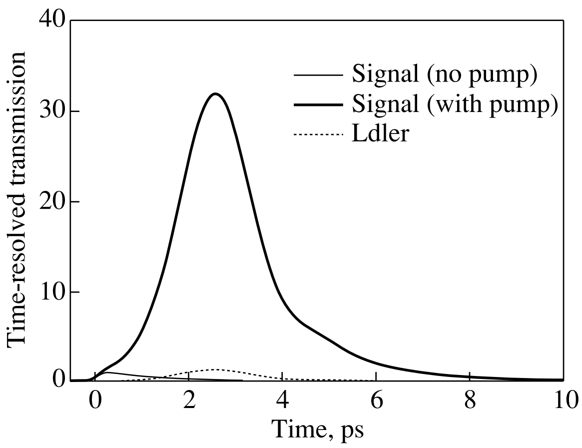

Fig. 4. Time-resolved transmission of the probe (with and without the presence of the pump beam) and additional emission (appearing in the presence of the pump beam) along the direction of the idler mode ($\theta \approx 47°$), showing the ultrafast dynamics of the parametric amplification. The sample is V_1, the temperature is 10 K, and the pump intensity is $I = I_0$. The exciting light pulses have a duration of 250 fs.

interference phenomena [15]. We have shown that the strong-coupling regime of SMCs, which alters the phase of the interacting waves, provides a means to control the interference process. The results here shown demonstrate that exciton–exciton collisions in semiconductors can be controlled and engineered. Moreover, they give precise indications of the production of almost decoherence-free collisions for the realization of completely optical microscopic switches and amplifiers. The availability of decoherence-free collisions appears crucial for the quantum control and manipulation of the polariton wave-function inside the cavity and for the realization of microscopic sources of nonclassical light. The possible control of exciton–exciton collisions opens new perspectives for the realization of entangled collective polariton states for quantum information and computation.

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